3D GEOMETRY

OBJECTIVES

1. **The direction cosines of the line which is perpendicular to the lines with direction cosines proportional to 6, 4, –4 and –6, 2, 1 is**

   a) 2, 3, 6 b) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ c) $\frac{2}{3},1,2$ d) $\frac{1}{3},\frac{3}{2},3$

2. **Angle between the lines whose direction cosine is given by $l + m + n = 0 = l^2 + m^2 - n^2$ is**

   a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{3}$

3. **The lines whose direction cosine are given by the relation $a^2l + b^2m + c^2n = 0$ and $mn + nl + lm = 0$ are parallel if**

   a) $(a^2 - b^2 + c^2)^2 = 4a^2c^2$ b) $(a^2 + b^2 + c^2)^2 = 4a^2c^2$

   c) $(a^2 - b^2 + c^2)^2 = a^2c^2$ d) None of these

4. **If a line makes the angle α, β, γ with the axes, the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to**

   a) 1 b) $\frac{5}{4}$ c) $\frac{3}{2}$ d) 2

5. **Consider the following statements**

   Assertion (A): The points A(5, –1, 1), B(7, –4, 7), C(1, –6, 10) and D(–1, –3, 4) are the vertices of a rhombus

   Reason (R): AB = BC = CD = DA

   a) Both A and R are true and R is the correct explanation of A

   b) Both A and R are true but A is not a correct explanation of A

   c) A is true but R is false

   d) A is false but R is true

6. **If a line makes angle $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$ is equal to**

   a) $\frac{4}{3}$ b) $\frac{8}{3}$ c) 2 d) 1

7. The point of intersection of the lines drawn from the vertices of any tetrahedron to the centroid of opposite faces divide the distance from each vertex to the opposite face in ratio

a) 4:3 b) 3:1 c) 2:1 d) 3:2

8. If vertices of tetrahedron are (1, 2, 3) (2, 3, 5), (3, –1, 2) and (2, 1, 4) then its centroid is

a) $\left(\frac{8}{3},\frac{5}{3},\frac{14}{3}\right)$ b) $\left(\frac{-8}{3},\frac{5}{3},\frac{14}{3}\right)$ c) $\left(2,5,\frac{7}{4}\right)$ d) $\left(2,\frac{5}{4},\frac{7}{2}\right)$

9. If P, Q, A, B are (1, 2, 5), (–2, 1, 3), (4, 4, 2) and (2, 1, –4) then the projection of PQ and AB is

a) $\frac{13}{4}$ b) 2 c) 3 d) 4

10. The projection of a line on the axes are 2, 3, 6 then the length of line is

a) 5 b) $2\sqrt{5}$ c) 7 d) Cannot determine

11. The distance of the point (–1, 2, 5) from the line which passes through (3, 4, 5) and whose direction cosines are proportional to 2, –3, 6 is

a) $\frac{4\sqrt{61}}{7}$ b) $\frac{2\sqrt{74}}{5}$ c) $\frac{2\sqrt{37}}{5}$ d) None of these

12. Let the coordinates of A, B, C are (1, 8, 4), (0, –11, 4), (2, –3, 1) respectively. The coordinate of a point D which is foot of the perpendicular from A on BC is

a) (3, 4, –2) b) (4, –2, 5) c) (4, 5, –2) d) (2, 4, 5)

13. The coordinates of A, B, C are A(–1, 2, –3), B(5, 0, –6), C(0, 4, –1). The direction cosines of the internal bisector of angle BAC are proportional to

a) 6, –2, 13 b) 21, 2, 2 c) 26, –4, 6 d) 25, 8, 5

14. Three lines with direction ratios 1, 1, 2; $\sqrt{3}-1, 4$ and $-\sqrt{3}-1, \sqrt{3}-1$, 4 make

a) A right angled triangle b) An Isosceles Triangle

c) An equilateral triangle d) None of these

15. In three dimensional geometry 2*x*+ 3 = 0 represents

a) A straight line parallel to y–axis b) a plane parallel to *yz* plane

c) A plane perpendicular to *yz* plane d) Either (a) or (b)

16. The equation to the plane passing through P(2, 6, 3) and at right angle to OP, where O is origin is

a) $2x + 6y + 3z + 49 = 0$ b) $2x + 6y + 3z = 49$

c) $2x + 6y + 3z = 47$ d) $2x + 6y + 3z + 47 = 0$

17. If acute angle between the planes $2x + ky + z = 6$ and $x + y + 2z = 3$ is $\frac{\pi}{3}$, then k equals

a) –1 b) 1 or 17 c) –1 or 17 d) –17

18. Equation of the plane passing through the points (0, 0, 1), (1, 0, 1) and (1, –1, 0) is

a) $x + y + z = 1$ b) $y + z = 1$ c) $x + y = 1$ d) $y - z + 1 = 0$

19. Equation of the plane through the point (4, 5, 1) and its normal is the line joining the points (3, 4, 2) and (1, 1, 1) is

a) $2x + 3y + z = 24$ b) $2x + 3y + z + 24 = 0$

c) $2x + 3y + z + 15 = 0$ d) $2x + 3y + z = 15$

20. The equation of the plane through the points (2, 2, 1) and (1, –2, 3) and parallel to the line joining the points (3, 2, –2) and (0, 6, –7) is

a) $12x + 11y - 16z + 14 = 0$ b) $12x - 11y - 16z - 14 = 0$

c) $12x + 11y - 16z - 14 = 0$ d) $12x - 11y - 16z + 14 = 0$

21. Equation of the plane through the points (2, 2, 1) and (1, –2, 3) and parallel to z–axis is

a) $2x + y = 2$ b) $2x - y = 2$ c) $2x - y + 2 = 0$ d) None of these

22. Equation of the plane that passes through point (–1, 1, –4) and is perpendicular to each of the planes $-2x + y + z = 0$ and $x + y - 3z + 1 = 0$ is

a) $4x + 5y + 3z = 11$ b) $4x - 5y - z = 11$

c) $4x - y - 3z = 11$ d) $4x + 5y + 3z + 11 = 0$

23. Equation of the plane passing through the point (1, 2, 3) and parallel to the plane $x + 2y + 3z + 4 = 0$ also passes through the point

a) (–4, –2, –3) b) (–4, 3, –2) c) (–4, 3, 2) d) (4, 3, 2)

24. Equation of the plane passing through the point (1, 2, 3) and perpendicular to the plane $x + 2y + 3z + 4 = 0$ must pass through the point

a) (1, 0, 1) b) (0, 0, 0) c) (0, 0, –1) d) (1, 0, –1)

25. Equation of the plane passes through the line of intersection of the planes $2x + y - 4 = 0$ and $y + 2z = 0$ and perpendicular to the plane $3x + 2y - 3z = 6$ is

a) $2x + 3y + 4z + 4 = 0$　　b) $2x - y - 4z - 4 = 0$

c) $2x + 3y + 4z - 4 = 0$　　d) $2x - y - 4z + 4 = 0$

26. The plane $x + y + z = 0$ is rotated through right angle about its line of intersection with the plane $2x + y + 4 = 0$ the equation of the plane in its new position is

a) $x - z + 4 = 0$　　b) $x + z + 4 = 0$　　c) $x - z + y = 4$　　d) $x - y = 4$

27. The equation of plane passing through the point (1, 1, 1), (1, –1, 1) and (–7, –3, –5) is perpendicular to

a) xy plane　　b) xz plane　　c) yz plane　　d) None of these

28. The equation of the plane passing through the point (–2, –2, 2) and containing the line joining the points (1, 1, 1) and (1, –1, 2) is

a) $x - 3y - 6z + 8 = 0$　　b) $x - 3y - 6z - 8 = 0$

c) $2x + 3y + 6z + 8 = 0$　　d) None of these

29. For what value of k points (–6, 3, 2), (3, –2, 4), (5, 7, 3) and (–13, k, –1) are coplanar

a) 13　　b) 15　　c) 0　　d) 17

30. The direction cosine of the perpendicular to a plane from origin are proportional to (3, 4, 5) and length of the perpendicular is $5\sqrt{2}$, the equation of the plane is

a) $3x + 4y + 5z = 1$　　b) $3x + 4y + 5z = 5\sqrt{2}$

c) $3x + 4y + 5z = 50$　　d) $3x + 4y + 5z = 25\sqrt{2}$

31. Equation of the plane which bisects at right angles to the join of (1, 3, –2) and (3, 1, 6) is

a) $x - y + 4z + 8 = 0$　　b) $x - y + 4z - 8 = 0$

c) $x - y + 4z - 12 = 0$　　d) $x - y - 4z + 12 = 0$

32. A variable plane passes through a fixed point (a, b, c) and meets the coordinate axis in A, B, C. The locus of midpoint of the plane common through A, B, C and parallel to the coordinate planes is

a) $ax^{-1} + by^{-1} + cz^{-1} = 1$　　b) $ax + by + cz = 1$

c) $ax^{-2} + by^{-2} + cz^{-2} = 1$　　d) None of these

33. Locus of the point, the sum of the square of whose distance from the planes $x + y + z = 0$, $x - z = 0$ and $x - 2y + z = 0$ is

a) $6x^2 + 4y^2 - 6z^2 + 3xy = 0$
b) $x^2 + y^2 + z^2 = 54$
c) $x^2 + y^2 + z^2 = 9$
d) $2(x^2 + y^2 + z^2) = 3$

34. Equation of bisector of acute angle between the planes $7x + 4y + 4z + 3 = 0$ and $2x + y + 2z + 2 = 0$ is

a) $x + y - 2z - 3 = 0$
b) $13x + 7y + 10z + 9 = 0$
c) $x + y - 2z + 3 = 0$
d) $13x + 7y + 10z - 9 = 0$

35. Distance between the parallel planes $2x - 2y + z + 3 = 0$ and $4x - 4y + 2z - 7 = 0$ is

a) $\frac{13}{12}$
b) $\frac{1}{6}$
c) $\frac{13}{6}$
d) $\frac{1}{12}$

36. Two system of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and p, q, r from the origin then

a) $a^{-2} + b^{-2} + c^{-2} = p^{-2} + q^{-2} + r^{-2}$
b) $a^2 + b^2 + c^2 = p^2 + q^2 + r^2$
c) $a + b + c = p + q + r$
d) $abc = pqr$

37. Area of the triangle whose vertices are (3, 4, –1), (2, 2, 1) and (3, –4, 3) is

a) $\sqrt{29}$
b) $\sqrt{32}$
c) 6
d) 7

38. Volume of the tetrahedron whose vertices are (2, 3, 2), (1, 1, 1), (3, –2, 1) and (7, 1, 4) is

a) $\frac{47}{6}$
b) $\frac{1}{2}$
c) $\frac{7}{2}$
d) None of these

39. The volume of the tetrahedron formed by the planes $x + y = 0$, $y + z = 0$, $z + x = 0$ and $x + y + z = 1$ is

a) 0
b) 1/6
c) 1/3
d) 2/3

40. Equation of the plane containing the line $3x + 4y + 6z - 3 = 0 = 2x - 4y + z + 6$ and passing through the origin is

a) $2x + 3y + 4z = 0$
b) $2x + y + 3z = 0$
c) $8x + 4y + 13z = 0$
d) None of these

41. The direction cosine of a line which are connected by the relation $l - 5m + 3n = 0$ and $7l^2 + 5m^2 - 3n^2 = 0$ are

a) $\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$ b) $\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}$ c) $\frac{1}{\sqrt{4}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ d) $\frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

42. In 3 dimensional geometry $ax + by + c = 0$ represents

a) A straight line on xy plane b) a plane parallel to z–axis

c) A plane perpendicular to z–axis d) a plane perpendicular to xz plane

43. If α, β, γ be the angles which a line makes with the positive direction of co-ordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

(a) 2 (b) 1

(c) 3 (d) 0

44. If the length of a vector be 21 and direction ratios be 2, – 3, 6 then its direction cosines are

(a) $\frac{2}{21}, \frac{-1}{7}, \frac{2}{7}$ (b) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$

(c) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (d) None of these

45. The point dividing the line joining the points $(1, 2, 3)$ and $(3, -5, 6)$ in the ratio $3 : -5$ is

(a) $\left(2, \frac{-25}{2}, \frac{3}{2}\right)$ (b) $\left(-2, \frac{25}{2}, \frac{-3}{2}\right)$

(c) $\left(2, \frac{25}{2}, \frac{3}{2}\right)$ (d) None of these

46. If the co-ordinates of the points *P* and *Q* be (1, –2, 1) and (2, 3, 4) and *O* be the origin, then

(a) $OP = OQ$ (b) $OP \perp OQ$

(c) $OP \parallel OQ$ (d) None of these

47. Distance of the point (1, 2, 3) from the co-ordinate axes are

(a) 13, 10, 5 (b) $\sqrt{13}, \sqrt{10}, \sqrt{5}$

(c) $\sqrt{5}, \sqrt{13}, \sqrt{10}$ (d) $\frac{1}{\sqrt{13}}, \frac{1}{\sqrt{10}}, \frac{1}{\sqrt{5}}$

48. If the points (–1, 3, 2), (–4, 2, –2) and $(5, 5, \lambda)$ are collinear, then $\lambda =$

(a) – 10 (b) 5

(c) – 5 (d) 10

49. The projections of a line on the co-ordinate axes are 4, 6, 12. The direction cosines of the line are

(a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (b) 2, 3, 6

(c) $\frac{2}{11}, \frac{3}{11}, \frac{6}{11}$ (d) None of these

50. *xy*-plane divides the line joining the points (2, 4, 5) and (– 4, 3, – 2) in the ratio

(a) 3 : 5 (b) 5 : 2

(c) 1 : 3 (d) 3 : 4

51. If the co-ordinates of *A* and *B* be (1, 2, 3) and (7, 8, 7), then the projections of the line segment *AB* on the co-ordinate axes are

(a) 6, 6, 4 (b) 4, 6, 4

(c) 3, 3, 2 (d) 2, 3, 2

52. If the centroid of triangle whose vertices are (*a*,1, 3), (– 2, *b*, –5) and (4, 7, *c*) be the origin, then the values of *a*, *b*, *c* are

(a) – 2, –8, –2 (b) 2, 8, –2

(c) –2, –8, 2 (d) 7, –1, 0

53. If a straight line in space is equally inclined to the co-ordinate axes, the cosine of its angle of inclination to any one of the axes

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

54. If α, β, γ be the direction angles of a vector and $\cos\alpha = \frac{14}{15}$, $\cos\beta = \frac{1}{3}$ then $\cos\gamma =$

(a) $\pm\frac{2}{15}$ (b) $\frac{1}{5}$

(c) $\pm\frac{1}{15}$ (d) None of these

55. The direction cosines of a line equally inclined to three mutually perpendicular lines having direction cosines as $l_1, m_1, n_1; l_2, m_2, n_2$ and l_3, m_3, n_3 are

(a) $l_1 + l_2 + l_3, m_1 + m_2 + m_3, n_1 + n_2 + n_3$ (b) $\frac{l_1 + l_2 + l_3}{\sqrt{3}}, \frac{m_1 + m_2 + m_3}{\sqrt{3}}, \frac{n_1 + n_2 + n_3}{\sqrt{3}}$

(c) $\frac{l_1 + l_2 + l_3}{3}, \frac{m_1 + m_2 + m_3}{3}, \frac{n_1 + n_2 + n_3}{3}$ (d) None of these

56. A line makes angles α, β, γ with the co-ordinate axes. If $\alpha + \beta = 90^o$, then $\gamma =$

(a) 0 (b) 90°

(c) 180° (d) None of these

57. If a line makes the angle α, β, γ with three dimensional co-ordinate axes respectively, then

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma =$

(a) – 2 (b) – 1

(c) 1 (d) 2

58. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two perpendicular lines, then the direction cosine of the line which is perpendicular to both the lines, will be

(a) $(m_1n_2 - m_2n_1), (n_1l_2 - n_2l_1), (l_1m_2 - l_2m_1)$

(b) $(l_1l_2 - m_1m_2), (m_1m_2 - n_1n_2), (n_1n_2 - l_1l_2)$

(c) $\frac{1}{\sqrt{l_1^2 + m_1^2 + n_1^2}}, \frac{1}{\sqrt{l_2^2 + m_2^2 + n_2^2}}, \frac{1}{\sqrt{3}}$

(d) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

59. The co-ordinates of a point *P* are (3, 12, 4) with respect to origin *O*, then the direction cosines of OP are

(a) 3, 12, 4 (b) $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$

(c) $\frac{3}{\sqrt{13}}, \frac{1}{\sqrt{13}}, \frac{2}{\sqrt{13}}$ (d) $\frac{3}{13}, \frac{12}{13}, \frac{4}{13}$

60. The locus of a first degree equation in x, y, z is a

(a) Straight line (b) Sphere

(c) Plane (d) None of these

61. The projection of a line on a co-ordinate axes are 2, 3, 6. Then the length of the line is

(a) 7 (b) 5

(c) 1 (d) 11

62. A line makes angles of $45°$ and $60°$ with the positive axes of *X* and *Y* respectively. The angle made by the same line with the positive axis of *Z*, is

(a) $30°$ Or $60°$ (b) $60°$ or $90°$

(c) $90°$ Or $120°$ (d) $60°$ or $120°$

63. If $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the direction cosines of a line, then the value of *n* is

(a) $\frac{\sqrt{23}}{6}$ (b) $\frac{23}{6}$

(c) $\frac{2}{3}$ (d) $\frac{3}{2}$

64. The direction cosines of the normal to the plane $x + 2y - 3z + 4 = 0$

(a) $-\frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (b) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (c) $-\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (d) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

65. The number of straight lines that are equally inclined to the three dimensional co-ordinate axes, is

(a) 2 (b) 4 (c) 6 (d) 8

66. If *O* is the origin and $OP = 3$ with direction ratios $-1, 2, -2$, then co-ordinates of *P* are

(a) (1, 2, 2) (b) $(-1, 2, -2)$ (c) (–3, 6, –9) (d) $(-1/3, 2/3, -2/3)$

67. If projection of any line on co-ordinate axis 3, 4, and 5, then its length is

(a) 12 (b) 50

(c) $5\sqrt{2}$ (d) $3\sqrt{2}$

68. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$ is

(a) $\frac{4}{3}$ (b) 1

(c) $\frac{8}{3}$ (d) $\frac{7}{3}$

69. If a line lies in the octant $OXYZ$ and it makes equal angles with the axes, then

(a) $l = m = n = \frac{1}{\sqrt{3}}$ (b) $l = m = n = \pm\frac{1}{\sqrt{3}}$

(c) $l = m = n = -\frac{1}{\sqrt{3}}$ (d) $l = m = n = \pm\frac{1}{\sqrt{2}}$

70. The equation of the plane passing through the point (–1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$, is

(a) $7x - 8y + 3z - 25 = 0$ (b) $7x - 8y + 3z + 25 = 0$

(c) $-7x + 8y - 3z + 5 = 0$ (d) $7x - 8y - 3z + 5 = 0$

71. If a plane cuts off intercepts $OA = a, OB = b, \ OC = c$ from the co-ordinate axes, then the area of the triangle ABC =

(a) $\frac{1}{2}\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$ (b) $\frac{1}{2}(bc + ca + ab)$

(c) $\frac{1}{2}abc$ (d) $\frac{1}{2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}$

72. A plane meets the co-ordinate axes in A, B, C and (α, β, γ) is the centered of the triangle ABC. Then the equation of the plane is

(a) $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$ (b) $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

(c) $\frac{3x}{\alpha} + \frac{3y}{\beta} + \frac{3z}{\gamma} = 1$ (d) $\alpha x + \beta y + \gamma z = 1$

73. Distance of the point (2,3,4) from the plane $3x - 6y + 2z + 11 = 0$ is

(a) 1 (b) 2

(c) 3 (d) 0

74. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$ meets the co-ordinate axes in A, B, C. The centroid of the triangle *ABC* is

(a) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ (b) $\left(\frac{3}{a}, \frac{3}{b}, \frac{3}{c}\right)$

(c) $\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$ (d) (a, b, c)

75. If O is the origin and A is the point (a, b, c) then the equation of the plane through A and at right angles to OA is

(a) $a(x-a)-b(y-b)-c(z-c)=0$

(b) $a(x+a)+b(y+b)+c(z+c)=0$

(c) $a(x-a)+b(y-b)+c(z-c)=0$

(d) None of these

76. The equation of the plane passing through the intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ the point (1, 1, 1), is

(a) $20x+23y+26z-69=0$

(b) $20x+23y+26z+69=0$

(c) $23x+20y+26z-69=0$

(d) None of these

77. The plane $ax+by+cz=1$ meets the co-ordinate axes in A, B and C. The centroid of the triangle is

(a) $(3a,3b,3c)$ (b) $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

(c) $\left(\frac{3}{a},\frac{3}{b},\frac{3}{c}\right)$ (d) $\left(\frac{1}{3a},\frac{1}{3b},\frac{1}{3c}\right)$

78. The equation of the plane through (1, 2, 3) and parallel to the plane $2x+3y-4z=0$ is

(a) $2x+3y+4z=4$ (b) $2x+3y+4z+4=0$

(c) $2x-3y+4z+4=0$ (d) $2x+3y-4z+4=0$

79. In the space the equation $by+cz+d=0$ represents a plane perpendicular to the plane

(a) YOZ (b) $Z=k$

(c) ZOX (d) XOY

80. A variable plane is at a constant distance p from the origin and meets the axes in A, B and C. The locus of the centroid of the tetrahedron $OABC$ is

(a) $x^{-2}+y^{-2}+z^{-2}=16p^{-2}$ (b) $x^{-2}+y^{-2}+z^{-2}=16p^{-1}$

(c) $x^{-2}+y^{-2}+z^{-2}=16$ (d) None of these

81. If the given planes $ax+by+cz+d=0$ and $a'x+b'y+c'z+d'=0$ be mutually perpendicular, then

(a) $\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}$ (b) $\frac{a}{a'}+\frac{b}{b'}+\frac{c}{c'}=0$

(c) $aa'+bb'+cc'+dd'=0$ (d) $aa'+bb'+cc'=0$

82. The points $A(-1,3,0)$, $B(2,2,1)$ and $C(1,1,3)$ determine a plane. The distance from the plane to the point $D(5,7,8)$ is

(a) $\sqrt{66}$ (b) $\sqrt{71}$

(c) $\sqrt{73}$ (d) $\sqrt{76}$

83. If *P* be the point (2, 6, 3), then the equation of the plane through *P* at right angle to *OP*, *O* being the origin, is

(a) $2x+6y+3z=7$ (b) $2x-6y+3z=7$

(c) $2x+6y-3z=49$ (d) $2x+6y+3z=49$

84. Distance between parallel planes $2x-2y+z+3=0$ and $4x-4y+2z+5=0$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) 2

85. The length of the perpendicular from the origin to the plane $3x+4y+12z=52$ is

(a) 3 (b) –4

(c) 5 (d) None of these

86. If the points $(1,1,k)$ and $(-3,0,1)$ be equidistant from the plane $3x+4y-12z+13=0$,then *k* =

(a) 0 (b) 1

(c) 2 (d) None of these

87. If a plane meets the co-ordinate axes at *A*,*B* and *C* such that the centroid of the triangle is (1, 2, 4) then the equation of the plane is

(a) $x+2y+4z=12$ (b) $4x+2y+z=12$

(c) $x+2y+4z=3$ (d) $4x+2y+z=3$

88. A plane π makes intercepts 3 and 4 respectively on z-axis and *x*-axis. If π is parallel to *y*-axis, then its equation is

(a) $3x+4z=12$ (b) $3z+4x=12$

(c) $3y+4z=12$ (d) $3z+4y=12$

89. Distance between two parallel planes $2x + y + 2z = 8$ **and** $4x + 2y + 4z + 5 = 0$ **is**

(a) $\frac{9}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) $\frac{3}{2}$

90. The angle between the planes $3x - 4y + 5z = 0$ **and** $2x - y - 2z = 5$ **is**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$ (d) None of these

91. The equation of the plane passing through (1, 1, 1) and (1, –1, –1) and perpendicular to $2x - y + z + 5 = 0$ **is**

(a) $2x + 5y + z - 8 = 0$ (b) $x + y - z - 1 = 0$

(c) $2x + 5y + z + 4 = 0$ (d) $x - y + z - 1 = 0$

3D GEOMETRY

HINTS AND SOLUTIONS

1. (b) Let *l*, *m*, *n* be the d.c. of required lines

Solving $6l + 4m - 4n = 0$ and $-6l + 2m + n = 0$ by cross multiplication, we have

$$\frac{l}{12} = \frac{m}{18} = \frac{n}{36} \text{ or } \frac{l}{2} = \frac{m}{3} = \frac{n}{6}$$

$$\therefore \text{ d.c. are } \frac{2}{\sqrt{2^2+3^2+6^2}}, \frac{3}{\sqrt{2^2+3^2+6^2}}, \frac{6}{\sqrt{2^2+3^2+6}}$$

2. (d) Given $l + m + n = 0$ ....(1)

$l^2 + m^2 - n^2 = 0$ ....(2)

Eliminating n from (1) & (2)

$l^2 + m^2 - (-l - m)^2 = 0$ or $lm = 0$

Either $l = 0$ or $m = 0$ when $l = 0$ from (1) & (2)

$m + n = 0$ or $m = n$

$m^2 = n^2 = 0$

$\therefore$ d.c. or one line is $0, -n, n$ or $(0, -1, 1)$ and when $m = 0$

d.r. of the second line is $(1, 0, -1)$

$$\therefore \cos\theta = \pm\left|\frac{1\times1-1\times0-1\times1}{\sqrt{0^2+(-1)^2+(1)^2}\sqrt{1^2+(0)^2+(-1)^2}}\right|$$

$$= \pm\frac{1}{2}, \theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

3. (a) Given relation $a^2l + b^2m + c^2n = 0$ ....(1)

$mn + nl + bn = 0$ ....(2)

Eliminating m from (1) and (2)

$$-\frac{1}{b^2}(a^2l + c^2n)n + nl - \frac{1}{b^2}(a^2l + c^2n)l = 0 \qquad ....(3)$$

Lines are parallel if roots of (3) are equal $(a^2 - b^2) + c^2) - 4a^2c^2 = 0$

4. (d) $\because$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ (sum of d.c.)

$\therefore$ $(1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

Or $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

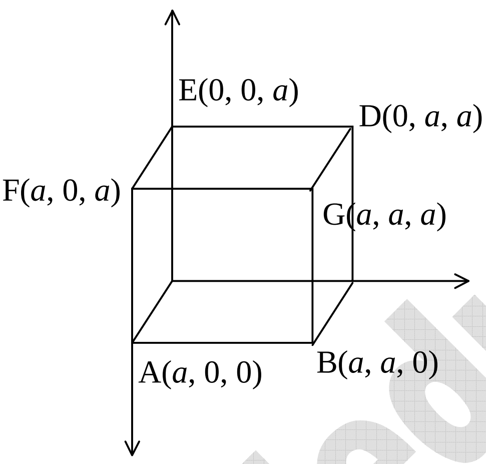
5. (a) By distance formula

AB = BC = CD = DA = 7

(Not that a square is also a rhombus)

6. (a) From figure OG, AD, BE and CF are four diagonals whose d.c. are

$$\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right),\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right),\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right) \text{ and } \left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$$



Let a line will d.c. l, m, n makes an angle α, β, γ, δ with the line OG, AD, BE and CF respectively.

Using $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$

We have $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$

$$= \left(\frac{l+m+n}{\sqrt{3}}\right)^2 + \left(\frac{-l+m+n}{\sqrt{3}}\right)^2 + \left(\frac{l+m-n}{\sqrt{3}}\right)^2 + \left(\frac{l-m-n}{\sqrt{3}}\right)^2$$

$$= \frac{4}{3}\left(l^2+m^2+n^2\right) = \frac{4}{3}$$

$$\therefore \sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = 4 - \frac{4}{3} = \frac{8}{3}$$

7. (b)

8. (d) centroid = $\left(\frac{\sum x_i}{4}, \frac{\sum y_i}{4}, \frac{\sum z_i}{4}\right)$

9. (c) d.r. of AB is 2, 3, 6. Its d.c. is $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

$\therefore$ Projection of PQ on AB = $\frac{2}{7}(1+2)+\frac{3}{7}(2-1)+\frac{6}{7}(5-3)=3$

10. (c) Let the length of the line is *r* and direction cosine of the line are *l, m, n*

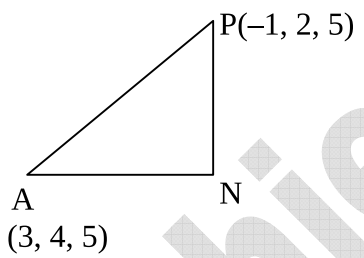
$\therefore$ $r\cos\alpha = 2 = rl$; $\quad r\cos\beta = 3 = rm$; $\ r\cos\gamma\ = 6 = rm$

$\therefore$ $(rl)^2 + (rm)^2 + (m)^2 = 2^2 + 3^2 + 6^2$

Or $r^2\ (l^2 + m^2 + n^2) = 49$

$\therefore$ $r = 7$

11. (a) d.c. of the line AN are $\frac{2}{7}, \frac{-3}{7}$ **and** $\frac{6}{7}$



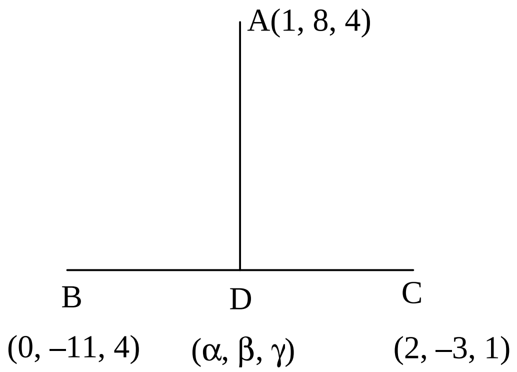
From figure $AP = \sqrt{(3+1)^2+(4-2)^2+(5-5)^2} = \sqrt{20}$

AN = projection of AP on the line = $\frac{2}{7}(3+1)+\frac{3}{7}(4-2)+\frac{6}{7}(5-5)\ = \frac{2}{7}$

$\therefore$ Required distance PN = $\sqrt{AP^2-(PN)^2}$

$= \sqrt{20-\frac{4}{49}}\ = \frac{4\sqrt{16}}{7}$

12. (c)



Let D ≡ (α, β, γ) since B, C, D lie on the same line

$\therefore \dfrac{\alpha - 0}{2-0} = \dfrac{\beta + 11}{-3+11} = \dfrac{\psi - 4}{1-4} = k$ (say)

$\therefore$ $\alpha = 2k$, $\beta = 8k - 11$, $\gamma -3k + 4$

Also AD is perpendicular to BC

$\therefore (2 - 0)\ (2k - 10 + (-3 + 11)\ (8k - 11 - 8) + (1 - 4)\ (4 + 3k - 4) = 0$

Or $k = 2$

$\therefore$ Point is $(2k, 8k - 11, -3k + 4) = (4, 5, -2)$

13. (d) d.r. of AB = 6, –2, –3

$\therefore$ d.c. of AB = $\dfrac{6}{7}, \dfrac{-2}{7}, \dfrac{-3}{7} = l_1, m_1, n_1$ (say)

Similarly d.c. of AC = $\dfrac{1}{3}, \dfrac{2}{3}, \dfrac{2}{3}$ = l_2, m_2, n_2 (say)

If θ be the angle between AB and AC, then d.c.of internal bisector is

$$\frac{l_1 + l_2}{2\cos\theta/2}, \frac{m_1 + m_2}{2\cos\theta/2} + \frac{n_1 + n_2}{2\cos\theta/2}$$

$\therefore$ d.r. of internal bisector is $l_1 + l_2 .m_1 + m_2$, $n_1 + n_2$

i.e., $\dfrac{25}{21}, \dfrac{8}{21}, \dfrac{5}{21}$ or 25, 8, 5

14. (c) Find d.r. of each side and then find the angle between two sides. Each angle is equal to

$\frac{\pi}{3}$

15. (b)

16. (b) d.r. of normal to the plane

$2-0,\ 6-0,\ 3-0 = 2, 6, 3$

Its equation is $2(x-2)+6(y-6)+3(z-3)=0$

Or $2x+6y+3z=49$

17. (c) d.r. of planes are 2, *k*, 1 and 1, 1, 2

$\therefore\ \cos\frac{\pi}{3}=\frac{2\times1+k\times1+1\times2}{\sqrt{4+k^2+1}\sqrt{1+1+4}}$ or $(5+k^2)6=\{2(k+4)\}^2$

Or $6k^2+30=4(k^2+8k+6)$ or $2k^2-32k-34=0$

$k^2-16k-17=0$

$(k-17)(k+1)=0$

$k=-1, 17$

18. (d) Equation of a plane through the point (0, 0, 1) is

$a(x-0)+b(y-0)+c(z-1)=0$

Or $ax+by+cz-c=0$

Since it passes through (1, 0, 1) and (1, –1, 0) then

$a=0,\ a-b-c=0$

Solving $a+0b+0c=0$

$a-b-c=0$

By cross multiplication we have $\frac{a}{0}=\frac{b}{+1}=\frac{c}{-1}$

$\therefore$ Reqd. equation is $+y-z+1=0$

Or $y-z+1=0$

19. (a) d.r. of normal to the plane is 3 –1, 4–1, 2–1 = 2, 3, 1

$\therefore$ Equation of plane is $2(x-4) + 3(y-5) + 1(z-1) = 0$ or $2x + 3y + z = 24$

20. (d) Any plane through (2, 2, 1) is $a(x-2) + b(y-2) + c(z-1) = 0$ ……………(1)

Since it passes through (1, –2, 3)

$a(1-2) + b(-2-2) + c(3-1) = 0$

Or $-a - 4b + 2c = 0$ ……………(2)

d.r. of the parallel line is 3, –4, 5

As a, b, c is the d.r. of normal to the plane which is parallel to the line with d.r. 3, –4, 5

$\therefore 3a - 4b + 5c = 0$ ……………(3)

On solving (2) & (3) we get $\frac{a}{12} = \frac{6}{-11} = \frac{c}{-16}$

$\therefore$ From (1), required equation is $12(x-2) - 11(y-2) - 15(z-1) = 0$

21. (b) Proceed same as 20, note that d.r. of z–axis is (0, 0, 1)

The normal to plane is perpendicular to z–axis $a.0 + b.0 + c = 0$ ……….(A)

Solve (A) with equation (2) of Q.N. 20 and put the value of a, b, c in (1) of Q.N. 20

22. (d) Any plane passing through (–1, 1, –4) is

$a(x+1) + b(y-1) + c(z+4) = 0$ ……………(1)

Since $-2x + y + z = 0$ & $x + y - 3z + 1 = 0$ are perpendicular to (1) then

$-2a + b + c = 0$ and $a + b - 3c = 0$

Solving $\frac{a}{-3-1} = \frac{b}{1-6} = \frac{c}{-2-1}$

Putting value of $(a, b, c) = (-4, -5, -3)$ in (1) we get the result.

23. (c) Equation of any plane parallel to $x + 2y - 3z + 4 = 0$ is $x + 2y - 3z = \lambda$

Since it passes through (1, 2, 3)

$1 + 4 - 9 = \lambda$ or $\lambda = -4$

$\therefore$ Equation of plane is $x + 2y - 3z + 4 = 0$ clearly (–4, 3, 2) satisfies it

24. (c) Let equation of plane be

$a(x-1) + b(y-2) + c(z-3) = 0$ …………….(1)

Since it is perpendicular to the plane

$x + 2y + 3z + 4 = 0$

$\therefore\ a.1 + b.2 + c.3 = 0$

$a = -(2b + 3c)$

From (1)

$-(2b + 3c)(x-1) + b(y-2) + c(z-3) = 0$

Or $b\{-2x + 2 + y - 2\} + c(-3x + 3 + z - 3) = 0$

$(y - 2x) + c/b(z - 3x) = 0$

Clearly it is always satisfied by (0, 0, 0)

25. (c) Any plane through line of intersection of the plane $2x + y - 4 = 0$ and $y + 2z = 0$ is

$(2x + y - 4) + \lambda(y + 2z) = 0$ ……………..(1)

d.r. of its normal are $2, 1 + \lambda, 2\lambda$

Since it is perpendicular to $3x + 2y - 3z = 6$

Hence $2 \times 3 + (1 + \lambda)\,2 + 2\lambda\,(-3) = 0$

Or $6 + 2 + 2\lambda - 6\lambda = 0$

Or $\lambda = 2$

Putting $\lambda = 2$ in (1) we get the required equation.

26. (a) Equation of a plane passing through the line of intersection of the plane

$x + y + z = 0$ and $2x + y + 4 = 0$ is

$(x + y + z) + k(2x + y + 4) = 0$ ………………(1)

If d.r. is $1 + 2k, 1 + k, 1$

d.r of $x + y + z = 0$ is 1, 1, 1

Since both are at right angles thus $1(1 + 2k) + (1 + k) + 1 \times 1 = 0$

Or $3k + 3 = 0$ or $k = -1$

Thus from (1) the required equation is $-x + z - 4 = 0$

27. (a) Equation of a plane passing through the point (–7, –3, –5) is

$a(x + 7) + b(y + 3) + c(x + 5) = 0$

Since it passes through the points (1, 1, 1) and (1, –1, 1)

$8a + 4b + 6c = 0$ and $8a - 4b + 6c = 0$

Solving we get $\frac{a}{48} = \frac{b}{0} = \frac{c}{-64}$

$\therefore$ d.r. of normal to the plane is 3, 0, –4 hence is perpendicular to xz plane

28. (a) Equation of any plane passing through (–2, –2, 2) is

$a(x + 2) + b(y + 2) + c(z - 2) = 0$ …………….91)

Since it contains the line joining the points (1, 1, 1) and (1, –1, 2) we get

$3a + 3b - c = 0$ and $3a + b + 0c = 0$

Solving by cross multiplication $\frac{a}{0+1} = \frac{b}{-3+0} = \frac{c}{3-9}$

Put $a = 1$, $b = -3$, $c = -6$ in (1) to get the required equation

29. (d) Four given points are coplanar it $\begin{vmatrix} -6 & 3 & 2 & 1 \\ 3 & -2 & 4 & 1 \\ 5 & 7 & 3 & 1 \\ -13 & k & -1 & 1 \end{vmatrix} = 0$

Or $\begin{vmatrix} -9 & 5 & -2 & 0 \\ -2 & -9 & 1 & 0 \\ 18 & 7-k & 4 & 1 \\ -13 & k & -1 & 1 \end{vmatrix} = 0$

$(R_1 \to R_1 \to R_2$

$R_2 \to R_2 - R_3$

$R_3 \to R_3 - R_4)$

Or $\begin{vmatrix} -9 & 5 & -2 \\ -2 & -9 & 1 \\ 0 & 17-k & 0 \end{vmatrix} = 0$ $(R_3 \to R_3 + 2R_1)$

Or – (17 – k) (–13) = 0   or k = 17

30. (c) d.c. of normal is

$$\frac{3}{\sqrt{3^2+4^2+5^2}},\frac{4}{\sqrt{3^2+4^2+5^2}},\frac{5}{\sqrt{3^2+4^2+5^2}} = \frac{3}{5\sqrt{2}},\frac{4}{5\sqrt{2}},\frac{5}{5\sqrt{2}}$$

Using $lx + my + nz = p$ the required equation is

$$\frac{3}{5\sqrt{2}}x+\frac{4}{5\sqrt{2}}y+\frac{5z}{5\sqrt{2}}=5\sqrt{2}$$

Or $3x + 4y + 5z = 50$

31. (b) d.r. of the line joining the points (1, 3, –2) and (3, 1, 6) is 2, –2, 8 or 1, –1, 4 it is also the d.r. of normal to the plane

Also plane passes through the points $\left(\frac{1+3}{2},\frac{3+1}{2},\frac{-2+6}{2}\right)$ is (2, 2, 2)

Thus its equation is $1(x-2)-1(y-2)+4(z-2)=0$

$x - y + 4z - 8 = 0$

32. (a) Let the equation of plane be $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

Where OA = α.OB = β and OC = γ

Since (1) passes through (a, b, c)

$\therefore\ \frac{a}{\alpha}+\frac{\beta}{\beta}+\frac{c}{\gamma}=1$ ……………(2)

The equation of the plane through A(α, 0, 0) and parallel to yz plane is $x = \alpha$. The equation of the plane passing through B(0, β, 0) and parallel to xz plane is $y = \beta$. The equation of the plane through C(0, 0, γ) and parallel to xy plane is $z = \gamma$

$\therefore$ Coordinate of the common point to the plane is (α, β, γ)

We have to find locus of α, β, γ which can be obtained by replacing (α, β, γ) by (x, y, z) in (2)

33. (c) Required locus is

$$\left(\frac{x+y+z}{\sqrt{3}}\right)^2+\left(\frac{x-z}{\sqrt{2}}\right)^3+\left(\frac{x-2y+z}{\sqrt{6}}\right)^2=9$$

On simplification, it gives $x^2 + y^2 + z^2 = 9$

34. (b) Equation of the planes bisecting the angle between the given planes are

$$\frac{7x+4y+4z+3}{\sqrt{7^2+4^2+4^2}}=\pm\frac{2x+y+2z+2}{\sqrt{2^2+1^2+2^2}}$$

Or $\dfrac{7x+4y+4z+3}{9}=\pm\dfrac{2x+y+2z+2}{3}$

Or $x + y - 2z - 3 = 0$, $13x + 7y + 10z + 9 = 0$

Let θ be the angle between $2x + y + 2z + 2 = 0$ and $x + y - 2z - 3 = 0$

$$\therefore \cos\theta=\frac{-2}{3}\left(\frac{1}{\sqrt{6}}\right)+\frac{-1}{3}\left(\frac{1}{\sqrt{6}}\right)+\left(-\frac{2}{3}\right)\left(-\frac{2}{\sqrt{6}}\right)=\frac{1}{3\sqrt{6}}$$

$\tan\theta=\sqrt{53}>1 \Rightarrow \theta = 45°$

$\therefore$ $x + y - 2z - 3 = 0$ is the bisector of obtuse angle, hence $13x + 7y + 10z + 9 = 0$ is the bisector of acute angle.

35. (c) The given planes are $4x - 4y + 2z + 6 = 0$ and $4x - 4y + 2z - 7 = 0$

Required distance $=\dfrac{6-(-7)}{\sqrt{4^2+4^2+2^2}}=\dfrac{13}{6}$

36. (a) Let the coordinate in two systems be (x, y, z) & (X, Y, Z) so that the equations of the plane in the two systems are

$\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ and $\dfrac{X}{p}+\dfrac{Y}{q}+\dfrac{Z}{r}=1$

Since origin is the same point in both system, the length of perpendicular from origin to both planes are equal i.e.

$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}+\frac{1}{r^2}}} \quad \text{Or} \quad a^{-2}+b^{-2}+c^{-2}=p^{-2}+q^{-2}+r^{-2}$$

37. (c) The vertices of the projection of the triangle on XY plane are (3, 4, 0), (2, 2, 0), (3, –4, 0).

$$\therefore\ \Delta xy = \frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ 2 & 2 & 1 \\ 3 & -4 & 1 \end{vmatrix} = \frac{1}{2}\times 8 = 4$$

Similarly $\Delta yz = \frac{1}{2}\begin{vmatrix} 4 & -1 & 1 \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix} = \frac{1}{2}\times 8 = 4$ and $\Delta xz = \frac{1}{2}\begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{vmatrix} = \frac{1}{2}|-4| = 2$

$\therefore$ Required area = $\sqrt{\Delta^2 xy + \Delta^2 yz + \Delta^2 zx} = \sqrt{4^2 + 4^2 + 2^2} = 6$

38. (c) For a tetrahedron

$$V = \frac{1}{6}\begin{vmatrix} 2 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 3 & -2 & 1 & 1 \\ 7 & 1 & 4 & 0 \end{vmatrix}$$

$$= \frac{1}{6}\begin{vmatrix} 1 & 2 & 1 & 0 \\ -2 & 3 & 0 & 0 \\ -4 & -3 & -3 & 0 \\ 7 & 1 & 4 & 1 \end{vmatrix} \qquad \begin{matrix} (R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \\ R_3 \to R_3 - R_4) \end{matrix}$$

$$= \frac{1}{6}\begin{vmatrix} 1 & 2 & 1 \\ -2 & 3 & 0 \\ -4 & -3 & -3 \end{vmatrix} = \frac{1}{6}|1(-9) - 2(-6) + 1(6+12)| = \frac{1}{6}|-3| = \frac{1}{2}$$

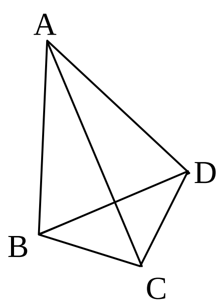
39. (d) Let plane ABC be $x + y = 0$

plane ACD be $y + z = 0$

plane ABD be $z + x = 0$

plane BCD be $x + y + z = 1$

solving three faces at a time we get the point of intersection as



A = (0, 0, 0)

B = (–1, 1, 1); C = (1, –1, 1); D = (1, 1, –1)

$$\therefore\ V = \frac{1}{6}\begin{vmatrix} 0 & 0 & 0 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{vmatrix} = \frac{1}{6}\begin{vmatrix} 0 & 2 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & 1 \end{vmatrix} \quad \begin{matrix}(C_1 \to C_1 + C_2 \\ C_2 \to C_2 + C_3)\end{matrix}$$

$$= \frac{1}{6}(2\times 2) = \frac{2}{3}$$

40. (c) Any plane passing through point of intersection of the plane

$3x + 4y + 6z - 3 = 0$ and $(2x - 4y + z + 6) = 0$ is

$(2x - 4y + z + 6) + \lambda(3x + 4y + 6z - 3) = 0$ ……………….(1)

Since if passes through origin $6 - 3\lambda = 0$

$\therefore$ Putting $\lambda = 2$ in (1) we get the required equation as $8x + 4y + 13z = 0$

41. (a) Given that $l - 5m + 3n = 0$……………..(1)

$7l^2 + 5m^2 - 3n^2 = 0$ ……………..(2)

Putting $l = 5m - 3n$ in (2) we get

$7(5m - 3n)^2 + 5m^2 - 3n^2 = 0$

Or $1800m^2 - 210mn + 60n^2 = 0$

Or $\frac{m}{n} = \frac{2}{3}, \frac{1}{2}$

When $\frac{m}{n} = \frac{2}{3}$, let $m = 2k$, $n = 3k$

$\therefore$ From (1), $l = 5m - 3n = k$

Also, $l^2 + m^2 + n^2 = 1$

$\Rightarrow k^2 + 4k^2 + 9k^2 = 1 \Rightarrow k = \pm\frac{1}{\sqrt{14}}$

Also $l, m, n = \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{2}{\sqrt{6}}$ or $\frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

Similarly when $\frac{m}{n}=\frac{1}{2}$

$\because l,m,n=\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}},\frac{-2}{\sqrt{6}}$ Or $\frac{-1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}}$

42. (b)

43. (a) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1 \Rightarrow \Sigma\sin^2\alpha=3-1=2$.

44. (b) D.c.'s are $\frac{2}{\sqrt{2^2+(-3)^2+6^2}},\frac{-3}{\sqrt{49}}$ and $\frac{6}{\sqrt{49}}$ **or** $\frac{2}{7},\frac{-3}{7},\frac{6}{7}$.

45. (b) $x=\frac{-5+9}{-2}=-2,\ y=\frac{-5(2)+3(-5)}{-2}=\frac{25}{2}$

$z=\frac{-5(3)+3(6)}{-2}=-\frac{3}{2}$.

46. (b) $a_1a_2+b_1b_2+c_1c_2=0$, so $OP\perp OQ$.

47. (b) From *x*-axis $=\sqrt{y^2+z^2}=\sqrt{4+9}=\sqrt{13}$

From *y*-axis $=\sqrt{1+9}=\sqrt{10}$

From *z*-axis $=\sqrt{1+4}=\sqrt{5}$.

48. (d) $\frac{-4+1}{5+4}=\frac{2-3}{5-2}=\frac{-2-2}{\lambda+2}$ **or** $\lambda+2=12$ **or** $\lambda=10$.

49. (a) Direction cosines $=\left(\frac{4}{\sqrt{16+36+144}},\frac{6}{14},\frac{12}{14}\right)$

or $\left(\frac{2}{7},\frac{3}{7},\frac{6}{7}\right)$.

50. (b) Required ratio $=-\left(\frac{5}{-2}\right)=\frac{5}{2}$ ***i.e.*, 5 : 2.**

51. (a) Here, $x_2-x_1=6$, $y_2-y_1=6$, $z_2-z_1=4$ **and d.c's of** x,y,z **-axes are (1,0,0), (0,1,0), (0, 0, 1) respectively.**

Now projection = $(x_2-x_1)l+(y_2-y_1)m+(z_2-z_1)n$

$\therefore$ Projections of line *AB* on co-ordinate axes are 6, 6, 4 respectively.

52. (c) $0=\frac{a-2+4}{3}\Rightarrow a=-2,\ 0=\frac{1+b+7}{3}\Rightarrow b=-8$

And $0=\frac{3-5+c}{3}\Rightarrow c=2$.

53. (c) Here, $\cos\alpha = \cos\beta = \cos\gamma$

$$\therefore\ 3\cos^2\alpha = 1 \Rightarrow \alpha = \cos^{-1}\left(\pm\frac{1}{\sqrt{3}}\right).$$

54. (a) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow \cos\gamma = \sqrt{1-\left(\frac{14}{15}\right)^2-\left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}-\left(\frac{196}{225}\right)} = \pm\frac{2}{15}.$$

55. (**b**) Standard Problem

56. b) Here, $\cos^2\alpha + \cos^2(90-\alpha) + \cos^2\gamma = 1$

$$\Rightarrow \cos^2\alpha + \sin^2\alpha + \cos^2\gamma = 1$$

$$\Rightarrow \cos^2\gamma + 1 = 1 \Rightarrow \gamma = 90^o.$$

57. (b) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$$

$$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3 = 2 - 3 = -1.$$

58. (a) Let lines are $l_1x + m_1y + n_1z + d = 0$ **.....(i)**

And $l_2x + m_2y + n_2z + d = 0$ .....(ii)

If $lx + my + nz + d = 0$ is perpendicular to (i) and (ii), then, $ll_1 + mm_1 + nn_1 = 0,\ ll_2 + mm_2 + nn_2 = 0$

$$\Rightarrow \frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - l_1n_2} = \frac{n}{l_1m_2 - l_2m_1} = d$$

Therefore, direction cosines are

$(m_1n_2 - m_2n_1),\ (n_1l_2 - l_1n_2), (l_1m_2 - l_2m_1)$.

59. (d) Required direction cosines are

$$\frac{3}{\sqrt{3^2+12^2+4^2}}, \frac{12}{\sqrt{3^2+12^2+4^2}}, \frac{4}{\sqrt{3^2+12^2+4^2}}$$

i.e., $\frac{3}{13}, \frac{12}{13}, \frac{4}{13}$.

60. (c) $Ax + By + Cz + D = 0$ **always represents a plane.**

61. (a) Let *d* be the length of line, then projection on *x*-axis = *dl* = 2, projection on *y*-axis = *dm* = 3, Projection on *z*-axis = *dn* = 6

Now $d^2(l^2 + m^2 + n^2) = 4 + 9 + 36$

$$\Rightarrow d^2(1) = 49 \Rightarrow d = 7.$$

62. (d) Given $\alpha = 45^o$, $\beta = 60^o$, $\gamma = ?$

$\because\ \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore\ \cos^2\gamma = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \Rightarrow \gamma = 60^o$ or 120^o.

63. (a) If $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ **are the d.c's of line then,** $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + n^2 = 1 \Rightarrow n^2 = \frac{23}{36} \Rightarrow n = \frac{\sqrt{23}}{6}$.

64. (a) The direction cosines of the normal to the plane are

$$\frac{1}{\sqrt{1^2+2^2+3^2}}, \frac{2}{\sqrt{1^2+2^2+3^2}}, \frac{-3}{\sqrt{1^2+2^2+3^2}}$$

i.e., $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}$.

But $x + 2y - 3z + 4 = 0$ can be written as $-x - 2y + 3z - 4 = 0$.

Thus the direction cosines are $\frac{-1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$.

65. (b) Since $\alpha = \beta = \gamma \Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \alpha = \cos^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$

So, there are four lines whose direction cosines are

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right),$$

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right).$$

66. (b) Co-ordinates of *P* are (lr, mr, nr)

Here $l = \frac{-1}{\sqrt{1^2+2^2+2^2}} = \frac{-1}{3}, m = \frac{2}{3}, n = \frac{-2}{3}$

And $r = 3$, (given)

$\therefore$ Co-ordinates of *P* are (–1, 2, –2).

67. (c) Let *d* be the length of line, then projection on *x*-axis = $dl = 3$, **projection on *y*-axis =** $dm = 4$ **and projection on *z*-axis =** $dn = 5$.

68.(c) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3} \Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = \frac{8}{3}$.

68. (b) Concept

69. (b) Given, equation of plane is passing through the point (–1, 3, 2)

$\therefore\ A(x+1)+B(y-3)+C(z-2)=0$ .....(i)

Since plane (i) is perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$

So, $A+2B+3C=0$ and $3A+3B+C=0$

71. (a) Length of sides are $\sqrt{a^2+b^2},\sqrt{b^2+c^2},\sqrt{c^2+a^2}$ respectively.

Now use $\Delta=\frac{1}{2}\sqrt{s(s-a)(s-b)(s-c)}$.

72. (a) Let the co-ordinates of the points where the plane cuts the axes are (*a*, 0, 0), (0, *b*, 0), (0, 0, *c*). Since centroid is (α,β,γ), therefore $a=3\alpha,\quad b=3\beta,\ c=3\gamma$.

Equation of the plane will be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$\Rightarrow \frac{x}{3\alpha}+\frac{y}{3\beta}+\frac{z}{3\gamma}=1\Rightarrow \frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3.$

73. (a) Required distance = $\left|\frac{6-18+8+11}{7}\right|=1$.

74. (d) Obviously, co-ordinates of *A*, *B*, *C* are respectively $(3a,0,0),\ (0,3b,0)$ **and** $(0,0,3c)$.

Hence centroid is (a,b,c).

75. (c) Normal will be *OA* whose direction ratios are $a-0,\ \ b-0,\ \ c-0$ ***i.e.*, *a*, *b*, *c*. It passes through** $A(a,b,c)$.

$\therefore$ Equation of required plane is,

$a(x-a)+b(y-b)+c(z-c)=0$

76. a) $(x+y+z-6)+\lambda(2x+3y+4z+5)=0\Rightarrow\lambda=\frac{3}{14}$

$\Rightarrow 20x+23y+26z-69=0$.

77. d) Centroid is $\left(\frac{\frac{1}{a}+0+0}{3},\frac{0+\frac{1}{b}+0}{3},\frac{0+0+\frac{1}{c}}{3}\right)$

i.e., $\left(\frac{1}{3a},\frac{1}{3b},\frac{1}{3c}\right)$.

78. (d) Plane parallel to the plane $2x+3y-4z=0$ **is** $2x+3y-4z+k=0$ **.....(i)**

Also plane (i) is passing through (1, 2, 3)

$\therefore\ (2)(1)+(3)(2)-(4)(3)+k=0 \Rightarrow k=4$

$\therefore$ Required plane is $2x+3y-4z+4=0$.

79. (a) The equation of *yz*-plane is $x = 0$.

i.e., $x+0.y+0.z=0$.

Clearly, given plane is perpendicular to *yz*-plane.

80. (a) Plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$**, where** $p=\frac{1}{\sqrt{\sum\left(\frac{1}{a^2}\right)}}$

Or $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$ .....(i)

Now according to equation, $x=\frac{a}{4},\ y=\frac{b}{4},\ z=\frac{c}{4}$

Put the values of *x*, *y*, *z* in (i), we get the locus of the centroid of the tetrahedron.

81. (d) It is a fundamental concept.

82. (a) Find the equation of the plane and find distance.

83. d) Distance of point *P* from origin $OP=\sqrt{4+36+9}=7$

Now d.r's of OP = 2–0, 6 – 0, 3 – 0 = 2, 6, 3

$\therefore$ d.c's of OP = $\frac{2}{7},\frac{6}{7},\frac{3}{7}$

$\therefore$Equation of plane in normal form is $lx+my+nz=p$

$\Rightarrow \frac{2}{7}x+\frac{6}{7}y+\frac{3}{7}z=7 \Rightarrow 2x+6y+3z=49$.

84. (c) The required distance is given by

$$\left|\frac{3}{\sqrt{2^2+2^2+1^2}}-\frac{5}{\sqrt{4^2+4^2+2^2}}\right|=\left|1-\frac{5}{6}\right|=\frac{1}{6}.$$

85. (d) $p=\left|\frac{-52}{\sqrt{9+16+144}}\right|=\left|\frac{-52}{13}\right|=|-4|$ **= 4.**

86. (b) $|3+4-12k+13|=|-9-12+13|$

$\therefore\ 3+4-12k+13=8 \Rightarrow k=1$.

87. (b) Given, plane meets the co-ordinate axes at $A(a,0,0),\ B(0,b,0)\ C(0,0,c)$

$\therefore$ Centroid $\equiv\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)=(1,2,4)$

$\Rightarrow \quad a = 3, b = 6, c = 12$

Hence, equation of required plane is, $\frac{x}{3}+\frac{y}{6}+\frac{z}{12}=1$

$\Rightarrow 4x + 2y + z = 12$.

88. (a) ***X*-intercept** $(a) = 4$; ***Z*-intercept** $(c) = 3$

Required equation = $\frac{x}{4}+\frac{z}{3}=1$ or $3x + 4z = 12$.

89. (c) Given planes are $2x + y + 2z - 8 = 0$

Or $4x + 2y + 4z - 16 = 0$ .....(i)

And $4x + 2y + 4z + 5 = 0$ .....(ii)

Distance between two parallel planes

$$= \left|\frac{-16-5}{\sqrt{4^2+2^2+4^2}}\right| = \frac{21}{6} = \frac{7}{2}.$$

90. (b) $\theta = \cos^{-1}\left[\frac{6+4-10}{\sqrt{50}\sqrt{9}}\right] = \cos^{-1}(0) = \frac{\pi}{2}$.

91. b) Any plane passing through (1, 1, 1) is $a(x-1)+b(y-1)+c(z-1)=0$